B.Sc. Semester-III Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 32118 **Course Code : SP/MTH/301/C-1C**

Course Title : Algebra

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meanings.

UNIT–I

1. Answer any **five** of the following questions: $2\times5=10$

a) Solve the equation $x^4+x^2-2x+6=0$, where $1+i$ is one root of the given equation.

b) If n is a positive integer, prove that $\frac{1.3.7...(2^n-1)}{2.4.8...2^n}<\frac{2^n}{2^{n+1}-1}$.

c) Find the product of all the values of $(1+i)^{\frac{4}{5}}$.

d) Give an example of an infinite set S and a mapping $F:S\to S$ such that F is surjective but not injective.

e) Show that the relation ρ defined on $\mathbb{R}$ by the rule "$x\rho y$ *iff* $x-y$ is irrational" is not an equivalence relation.

f) Let $f:A\to B$ and $g:B\to C$ be two invertible functions. Then show that $g\circ f$ is invertible.

g) Determine the rank of the matrix A^3+A^2+A, where

$$A=\begin{bmatrix}2&0&1\\3&3&0\\6&2&3\end{bmatrix}.$$

h) Prove that n^2-2 is not divisible by 4, $n\in Z$.

UNIT–II

2. Answer any **four** of the following questions: $5\times4=20$

a) Use De Moiver's theorem to prove that

$$\tan 5\theta=\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}.$$

b) Solve the equation : $x^4+x^3-2x^2+4x-24=0$.

c) Suppose A is a 2×2 real matrix with trace 5 and determinant 6. Find the eigenvalues of the matrix $B=A^2-2A+I_2$.

d) Find a linear operator T on $\mathbb{R}^3$ such that *Ker T* is the subspace $U = \{(x, y, z) \in R^3 : x + y + z = 0\}$ of $\mathbb{R}^3$.

e) i) Prove that the rank of a real skew-symmetric matrix cannot be 1.

ii) If gcd(m, n)=1, prove that gcd(mn, m+n)=1, where m and n are positive integers. 3+2

f) For what values of k the following system of equations has a non-trivial solution? Solve in any one case.

$x + 2y + 3z = kx$; $2x + y + 3z = ky$; $2x + 3y + z = kz$

UNIT–III

3. Answer any **one** of the following questions:

10×1=10

a) i) Prove that two eigenvectors of a square matrix A over the field $\mathbb{R}$ corresponding to two distinct eigen values of A are linearly independent.

ii) Find the sum of 99th powers of the roots of the equation $x^7 - 1 = 0$.

iii) Find the least positive residue of 3^{36} (*mod* 77). 4+3+3=10

b) i) Show that S = {(2, –5, 3)} is not a subspace of $V_3(\mathbb{R})$ generated by the vectors (1,–3, 2), (2, –4, 1), (1, –5, 7).

ii) Show that the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ is not diagonalizable.

iii) Solve the equation:

$$x^6 - x^5 + x^4 - 2x^3 + x^2 - x + 1 = 0.$$

3+3+4=10

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